

A Renormalization Group Study of the $(\phi^*\phi)^3$ Model coupled to a Chern-Simons Field*

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(October 7, 1999)

Abstract

We consider the model of a massless charged scalar field, in $(2+1)$ dimensions, with a self interaction of the form $\lambda(\phi^*\phi)^3$ and interacting with a Chern Simons field. We calculate the renormalization group β functions of the coupling constants and the anomalous dimensions γ of the basic fields. We show that the interaction with the Chern Simons field implies in a β_λ which suggests that a dynamical symmetry breakdown occurs. We also study the effect of the Chern Simons field on the anomalous dimensions of the composite operators $(\phi^*\phi)^n$, getting the result that their operator dimensions are lowered.

PACS numbers: 11.10.Gh, 11.10.Hi, 11.10.Kk

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I. INTRODUCTION

Self interacting scalar fields are the simplest nontrivial field theories. Nevertheless they have found large application in many different phenomena. Renormalization group analyses of the model of scalar fields in $(2 + 1)$ dimensions, with a self interaction of the form $\lambda\phi^6$ have appeared in the literature [1] in conjunction with other self interactions, and also in interaction with other fields. On the other side, the Chern-Simons (CS) field theory [2] is known to cause some strange effects in matter fields, the most known being the transmutation of their spins and statistics [3].

Bosons (fermions) interacting with a CS field get an extra contribution to their spins and statistical phases, changing to anyons and even to fermions (bosons). Studies of the change in the scale behavior of matter fields due to their interaction with the CS field have also been considered [4–6].

In this paper we study the model of a massless charged scalar field with a self interaction of the form $\lambda(\phi^*\phi)^3$ and interacting with an Abelian CS field. Classically it only involves dimensionless parameters and is scale invariant. It is also strictly renormalizable: no induction of terms of the forms $m^2(\phi^*\phi)$ or $g(\phi^*\phi)^2$ occurs. Besides the calculation of the anomalous dimensions of ϕ and A^μ and the β functions related to their coupling constants, we also calculate the anomalous dimensions of composite operators of the form $(\phi^*\phi)^n$. Some of our conclusions agree and others disagree with the previous literature. This will be discussed in section III and in the Conclusions.

To regulate the ultraviolet (UV) behavior we use a simplified version of dimensional regularization, the so called “Dimensional Reduction” method. It consists of contracting and simplifying the Lorentz tensors, before extending the Feynman integrals out of 3 dimensions. This procedure, previously used by several authors [4–6], greatly simplifies calculations involving the CS field, because it does not require the extension of the Levi-Civita tensor $\varepsilon^{\mu\nu\rho}$ out of 3 dimensions. In Feynman integrals only involving scalar vertices and propagators, no difference appear between the results gotten by using one or the other method. In graphs involving the CS field and the $\varepsilon^{\mu\nu\rho}$, the differences of this method to a “full” dimensional regularization would only show up [4] in sub-leading contributions to the Feynman integrals; that is, if D stands for the extended dimension of the space time when the Feynman integrals are expanded in Laurent series in $\epsilon \equiv (D - 3)$, no difference in the leading divergent term in $1/\epsilon$ will appear. It is, on the other side, a characteristic of dimensional regularization in $(2 + 1)$ dimensions, that one loop graphs are finite, and 2 loops graphs have at most a single pole divergence in ϵ . As the calculation of the renormalization group parameters only involve the use of the divergent parts of the graphs, no differences to the full dimensional regularization is expected up to 2 loops in graphs that involve the CS propagator, and any number of loops in graphs only involving the scalar propagator. In this paper we will restrict the calculations to up 2 loops in all graphs involving propagators of the CS field, and 4 loops in graphs involving only the propagator of the scalar field. As we will explicitly show that (at least) to that orders, dimensional reduction is enough to regularize the model and to preserve the gauge symmetry, as expressed by the Ward Identities (WI). We will work in the Landau gauge and, avoiding exceptional momenta, no infrared (IR) divergences appear.

The plan of the paper is as follows. In section II the model is presented, and the divergent UV counterterms, necessary for the renormalization group study, are obtained by

calculating the CS 2 point function and the scalar field 2 and 6 point functions. In section III the renormalization group β functions and anomalous dimensions of the fields are obtained, and compared with other calculations. The change in the dynamical behavior of the ϕ field due to the interaction of the CS field is discussed. The influence of the CS in the dimension and renormalizability of operators of the form $(\phi^*\phi)^n$ is also studied. A summary of the results are presented in the Conclusions. In Appendix A the explicit verification of the WI is given, and in Appendix B some Feynman integrals are calculated as examples.

II. THE MODEL

The model is constituted by a massless charged boson in $2 + 1$ dimensions represented by a field ϕ , with a self interaction of the form $(\phi^*\phi)^3$ and minimally interacting with a Chern-Simons (CS) field A_μ . Its Lagrangian density is given by

$$\begin{aligned} \mathcal{L} = & \partial_\mu \phi_0^\dagger \partial^\mu \phi_0 - ie_0 A_0^\mu (\phi_0^\dagger \partial_\mu \phi_0 - \partial_\mu \phi_0^\dagger \phi_0) + e_0^2 A_0^\mu A_{0\mu} (\phi_0^\dagger \phi_0) \\ & - \frac{\lambda_0}{6^2} (\phi_0^\dagger \phi_0)^3 + \frac{1}{2} \varepsilon_{\mu\nu\rho} A_0^\mu \partial^\nu A_0^\rho. \end{aligned} \quad (2.1)$$

The metric is $g_{\mu\nu} = (1, -1, -1)$, ∂_μ stands for $\frac{\partial}{\partial x^\mu}$, $\varepsilon_{\mu\nu\rho}$ is the antisymmetric Levi-Civita tensor with $\varepsilon^{012} = 1$, and e_0 and λ_0 are dimensionless coupling constants. The subscript “0” means that the corresponding quantity is “unrenormalized”.

The model is renormalizable: all the UV infinities of the perturbative series can be absorbed in a redefinition of the unrenormalized quantities. It also has a gauge symmetry what suggests the use of dimensional regularization [7]. However, the presence of the Levi-Civita tensor in the CS term makes dimensional regularization cumbersome and the calculations become awkward in more than one loop. We will so, take advantage of some characteristics of $(2 + 1)$ dimensions and use a simplified version of Dimensional Regularization, the so called Dimensional Reduction [4,5]. In this procedure, the Lorentz tensor algebra is considered in $(2 + 1)$ dimensions and only the remaining scalar Feynman integrals are extended out of $(2 + 1)$ dimensions. It was verified in [5], up to 2 loops, that for the non-Abelian Chern-Simons theory, this procedure, in fact preserves the Slavnov-Taylor identities. As we will also show below up to 2 loops, it also preserves the Ward identities in our model, and no inconsistencies appear.

To get information on the asymptotic behavior of the model, we need to calculate the renormalization group parameters: β functions and anomalous dimensions of the fields. For this task, adopting the Renormalization Group approach of t’Hooft [8] based on minimal subtraction, we only need to calculate the divergent parts of some vertex functions, more precisely the residues of the poles in $1/\epsilon$, where $\epsilon = 3 - D$ and D is the “extended” dimension of the space time. In $(2 + 1)$ dimension this means that we must go to at least 2 loops calculations, because as a characteristic of dimensional regularization, one loop integrals are finite.

Introducing the renormalized fields ϕ and A_μ and the renormalized coupling constants e and λ through the definitions

$$\phi_0 = Z_\phi^{\frac{1}{2}} \phi = (1 + A)^{\frac{1}{2}} \phi \quad (2.2)$$

$$A_0^\mu = Z_A^{\frac{1}{2}} A^\mu = (1 + B)^{\frac{1}{2}} A^\mu \quad (2.3)$$

$$e_0 = e\mu^{\frac{\epsilon}{2}}(1 + D)/Z_\phi Z_A^{\frac{1}{2}} \quad (2.4)$$

$$e_0^2 = e^2\mu^\epsilon(1 + E)/Z_\phi Z_A \quad (2.5)$$

$$\lambda_0 = \mu^{2\epsilon}(\lambda + C)/Z_\phi^3 \quad (2.6)$$

where μ is a mass parameter introduced to keep e and λ dimensionless quantities, and A to E are the counterterms to be chosen so as to make the renormalized quantities finite, in each order of perturbation. As will be seen in the calculations, the renormalization of λ in presence of the CS field is not multiplicative. By substituting these definitions in (2.1) we get for \mathcal{L} :

$$\begin{aligned} \mathcal{L} = & \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{2} \varepsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho - ie\mu^{\frac{\epsilon}{2}} A^\mu (\phi^\dagger \partial_\mu \phi - \partial_\mu \phi^\dagger \phi) + e^2 \mu^\epsilon A^\mu A_\mu (\phi^\dagger \phi) \\ & - \frac{\lambda \mu^{2\epsilon}}{6^2} (\phi^\dagger \phi)^3 + A \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{B}{2} \varepsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho - ie\mu^{\frac{\epsilon}{2}} D A^\mu (\phi^\dagger \partial_\mu \phi - \partial_\mu \phi^\dagger \phi) \\ & + e^2 \mu^\epsilon E A^\mu A_\mu (\phi^\dagger \phi) - \frac{\mu^{2\epsilon} C}{6^2} (\phi^\dagger \phi)^3. \end{aligned} \quad (2.7)$$

The Feynman rules for this Lagrangian in the Landau gauge are depicted in figure 1. This gauge can be implemented by adding to the Lagrangian a gauge fixing term: $\frac{1}{2\xi}(\partial^\mu A_\mu)^2$, inverting the free quadratic part of the A^μ to get the CS propagator and, then letting $\xi \rightarrow \infty$. The would be Faddeev-Popov ghost field is completely decoupled of the other fields and does not have any effect. Calling $\Gamma(p)$ the scalar field 1PI two point function, and $\Gamma^\mu(q; p, p')$ and $\Gamma^{\mu\nu}(q, k; p, p')$, respectively the trilinear and quadrilinear CS scalar field vertices, where q and k represent “photon” momenta and p and p' scalar field momenta, we have the WI

$$q^\mu \Gamma_\mu(q; p, p') = -e[\Gamma(p') - \Gamma(p)] \quad (2.8)$$

$$q^\mu \Gamma_{\mu\nu}(q, k; p, p') = -e[\Gamma_\nu(k; p + q, p') - \Gamma_\mu(k; p, p' - q)], \quad (2.9)$$

which require that $E = D = A$, leaving us with only three (we choose A, B, C) counterterms to be fixed. An explicit proof of these WI in two loops is given in the Appendix A.

To determine A, B , and C we need to calculate the simple pole part of the 2 point function of the CS field, $\Pi_{\mu\nu}(q)$, and the scalar field 2 and 6 point functions, respectively Γ_2 and Γ_6 . In graphs involving the CS field, we will extend the calculations up to two loops getting at most a simple pole in $1/\epsilon$; in graphs only involving the scalar field we will go up to four loops. So, in the tensorial Feynman integrals, in which dimensional reduction could possibly differ from dimensional regularization (in the sub leading terms in $1/\epsilon$) no difference between the two methods are expected in the calculation of the counterterms and in the renormalization group parameters.

Let us start with $\Pi_{\mu\nu}$. The only divergent diagrams, up to 2 loops, are those shown in figure 2 (the possible counterterm is also drawn in the figure). Their contributions are given by

$$(2a) = 4e^4 \int \mathcal{D}q \int \mathcal{D}k \frac{\varepsilon_{\mu\nu\rho} k^\rho}{k^2(q-p)^2(q+k)^2} \quad (2.10)$$

$$(2b) = e^4 \int \mathcal{D}q \int \mathcal{D}k \frac{(2k+q)^\alpha \varepsilon_{\alpha\beta\gamma} q^\gamma (2k+q-2p)^\beta (2k-p)_\mu (2k+2q-p)_\nu}{k^2(k+q)^2 q^2 (k-p)^2 (q+k-p)^2}, \quad (2.11)$$

where $\mathcal{D}q \equiv \mu^\epsilon d^{3-\epsilon} q / (2\pi)^{3-\epsilon}$ and an infinitesimal imaginary part is supposed in every propagator denominator ($p^2 \rightarrow p^2 + i\eta$, $\eta \ll 1$). Both integrals are logarithmically divergent. The divergent parts are of the form $\varepsilon_{\mu\nu\rho} p^\rho I$ where I is a scalar integral, that can be calculated by the usual dimensional continuation, after reducing the denominator to a single monomial through the use of Feynman parameters. The results are

$$(2a) = 4\varepsilon_{\mu\nu\rho} p^\rho \left(-\frac{e^4}{96\pi^2} \frac{1}{\epsilon} \right) + \text{finite part} \quad (2.12)$$

$$(2b) = \varepsilon_{\mu\nu\rho} p^\rho \left(\frac{e^4}{24\pi^2} \frac{1}{\epsilon} \right) + \text{finite part}. \quad (2.13)$$

As can be seen, the divergent parts of the two integrals cancel each other and we are left with only finite contributions to $\Pi_{\mu\nu}$. So, the counterterm B can be chosen as $B = 0$, and no infinite wave function renormalization of the CS field interacting with massless scalar field is needed. This result extends for massless matter, the result of the Coleman-Hill theorem [9].

Let us now look at the scalar two point function, $\Gamma_2(p)$. The divergent graphs up to second order in α and λ are shown in figure 3, together with the counterterm. Their contributions are given by

$$(3a) = -2e^4 i \int \mathcal{D}q \frac{1}{(p+q)^2} \int \mathcal{D}k \varepsilon_{\mu\nu\rho} \frac{k^\rho}{k^2} \varepsilon^{\nu\mu\gamma} \frac{(k+q)_\gamma}{(k+q)^2} \quad (2.14)$$

$$(3b) = e^4 i^3 \int \mathcal{D}q \varepsilon_{\alpha\beta\gamma} \frac{q^\gamma}{q^2} \frac{(2p+q)^\beta}{(p+q)^2} \int \mathcal{D}k \frac{(2p+k)^\mu (2p+2q+k)^\nu (2k+2p+q)^\alpha}{(p+k)^2 (p+q+k)^2} \varepsilon_{\mu\nu\rho} \frac{k^\rho}{k^2} \quad (2.15)$$

$$(3c) = e^4 i^3 \int \mathcal{D}q \frac{1}{(p+q)^2} \varepsilon_{\mu\alpha\lambda} \frac{q^\lambda}{q^2} \varepsilon_{\beta\nu\rho} \frac{q^\rho}{q^2} \int \mathcal{D}k \frac{(2k+q)^\alpha (2k+q)^\beta (2p+q)^\mu (2p+q)^\nu}{k^2 (k+q)^2} \quad (2.16)$$

$$(3d) = -\frac{\lambda^2}{2^2 \cdot 3} i^5 \int \mathcal{D}k_1 \mathcal{D}k_2 \mathcal{D}k_3 \mathcal{D}k_4 \frac{1}{k_1^2 k_2^2 k_3^2 k_4^2 (p+k_1+k_2-k_3-k_4)^2}. \quad (2.17)$$

After the simplification of the tensor algebra in $(2 + 1)$ dimensions we are left with multiple scalar integrals that can be made, one loop at time, through the reduction of the denominators by successive use of Feynman parameters. The results are

$$(3a) = -2ie^4 \left(\frac{p^2}{96\pi^2} \frac{1}{\epsilon} + \dots \right) \quad (2.18)$$

$$(3b) = -ie^4 \left(\frac{p^2}{12\pi^2} \frac{1}{\epsilon} + \dots \right) \quad (2.19)$$

$$(3c) = -ie^4 \left(\frac{p^2}{24\pi^2} \frac{1}{\epsilon} + \dots \right) \quad (2.20)$$

$$(3d) = -i \frac{\lambda^2}{2^2 \cdot 3} \left(-\frac{p^2}{3 \cdot 2^{11} \pi^4} \frac{1}{\epsilon} + \dots \right) . \quad (2.21)$$

For the contribution (iAp^2) of the counterterm to cancel these divergences we must choose:

$$A = \left(\frac{7}{48\pi^2} \alpha^2 - \frac{1}{3^2 \cdot 2^{13} \pi^4} \lambda^2 \right) \frac{1}{\epsilon} . \quad (2.22)$$

Let us now proceed to the calculation of C , the counterterm of the coupling constant λ . For this task we need to get the divergent parts of $\Gamma_6(p_1, \dots, p_6)$. After a lengthy analyses of the many graphs involved, we are left with the divergent contributions drawn in figure 4. The bullets on the diagrams $4p, 4q, 4r, 4s$ and $4t$ mean the insertion of the counterterm in the corresponding vertex. The calculation of all these diagrams can be reduced to the calculation of the nine integrals represented in figure 5. In the Appendix B we show, as examples, the calculation (of the divergent parts) of $5a, 5b, 5d$ and $5f$. Here we present only the results:

$$\mathcal{G}(p, q) = -\frac{1}{2^5 \pi^2} \frac{1}{\epsilon} + \text{finite part}, \quad (2.23)$$

$$\mathcal{H}(p) = \frac{i}{16\pi^2} \frac{1}{\epsilon} + \text{finite part}, \quad (2.24)$$

$$\Delta_3(p) = -\frac{i}{2^5 \pi^2} \left[\frac{1}{\epsilon} + \left(\log \frac{4\pi\mu^2}{-p^2} - 3 - 2\gamma - 2\log 2 \right) + \mathcal{O}(\epsilon) \right], \quad (2.25)$$

$$\mathcal{Y}(p) = -\frac{1}{2^{12} \pi^4} \frac{1}{\epsilon} + \text{finite part}, \quad (2.26)$$

$$\mathcal{Z}(p, q) = \frac{1}{2^{11} \pi^4} \frac{1}{\epsilon} + \text{finite part}, \quad (2.27)$$

$$\mathcal{W}(q, p) = -\frac{1}{2^{11}\pi^4} \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(2 \log \frac{4\pi\mu^2}{-(p+q)^2} + 8 - \frac{11}{2}\gamma \right) + \text{finite part} \right], \quad (2.28)$$

$$\mathcal{M}(a, c, d) = \frac{3i}{2^6\pi^2} \frac{1}{\epsilon} + \text{finite part}, \quad (2.29)$$

$$\mathcal{N}(a, c, d) = \frac{i}{2^5\pi^2} \frac{1}{\epsilon} + \text{finite part}, \quad (2.30)$$

and

$$\mathcal{Q}(a, b, c) = \frac{1}{2^5\pi^2} \frac{1}{\epsilon} + \text{finite part} \quad (2.31)$$

where γ is the Euler constant. In some graphs we will need the result of $\Delta_3^2(p)$:

$$\Delta_3^2(p) = -\frac{1}{2^{10}\pi^4} \left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left(2 \log \frac{4\pi\mu^2}{-p^2} - 2(3 + 2\gamma + 2 \log 2) \right) + \text{finite part} \right]. \quad (2.32)$$

By collecting all contributions of figure 4 we can write

$$\begin{aligned} & \Gamma_6(p_1, p_2, p_3, p'_1, p'_2, p'_3) \mu^{-2\epsilon} \\ &= -\frac{\lambda^2}{6} \Delta_3(p_1 + p_2 + p_3) - \frac{\lambda^2}{2} [\Delta_3(p_1 + p_2 - p'_2) + 8 \text{ terms}] \\ &+ 2i\lambda\alpha^2 [\mathcal{G}(p_1, -p'_1) + 8 \text{ terms}] + 2i\lambda\alpha^2 [\mathcal{G}(p_1, p_2) + 2 \text{ terms}] \\ &+ 2i\lambda\alpha^2 [\mathcal{G}(-p'_1, -p'_2) + 2 \text{ terms}] - 2\lambda\alpha^2 [\mathcal{H}(p_1 - p'_1) + 8 \text{ terms}] \\ &+ i\frac{5}{4}\lambda^3 [\mathcal{Y}(p_1 - p'_1, p_2 - p'_2) + 5 \text{ terms}] \\ &+ i\frac{3}{4}\lambda^3 [\mathcal{Y}(p_1 + p_2, -(p_1 + p'_2)) + 8 \text{ terms}] \\ &+ i\frac{1}{4}\lambda^3 [\mathcal{Z}(p_1, p_2) + 2 \text{ terms}] - \frac{5}{12}\lambda^2 [\mathcal{Z}(p_1, -p'_1) + 8 \text{ terms}] \\ &+ i\frac{1}{4}\lambda^3 [\mathcal{Z}(-p'_1, -p'_2) + 2 \text{ terms}] \\ &+ i\frac{1}{36}\lambda^3 [\Delta_3^2(p_1 + p_2 + p_3)] + i\frac{1}{4}\lambda^3 [\Delta_3^2(p_1 + p_2 - p'_1) + 8 \text{ terms}] \\ &+ i\frac{1}{4}\lambda^3 [\mathcal{W}(p_1, p_2 + p_3) + 2 \text{ terms}] \\ &+ i\frac{1}{4}\lambda^3 [\mathcal{W}(-p'_1, -(p'_2 + p'_3)) + 2 \text{ terms}] \\ &+ i\frac{3}{4}\lambda^3 [\mathcal{W}(p_1, p_2 - p'_1) + 17 \text{ terms}] \\ &+ i\frac{3}{4}\lambda^3 [\mathcal{W}(-p'_1, p_1 - p'_2) + 17 \text{ terms}] \\ &+ i\frac{7}{12}\lambda^3 [\mathcal{W}(p_1, -(p'_1 + p'_2)) + 8 \text{ terms}] \end{aligned}$$

$$\begin{aligned}
& + i \frac{7}{12} \lambda^3 [\mathcal{W}(-p'_1, p_1 + p_2) + 8 \text{ terms}] - \frac{\lambda C}{3} \Delta_3(p_1 + p_2 + p_3) \\
& - \lambda C [\Delta_3(p_1 + p_2 - p'_1) + 8 \text{ terms}] - iC \\
& + 2^4 \alpha^4 [\mathcal{M}(p_1, p_2 - p'_2, p_3 - p'_3) + 17 \text{ terms}] \\
& + 2^5 \alpha^4 [\mathcal{N}(p_1, p_2 - p'_2, p_3 - p'_3) + 17 \text{ terms}] \\
& + i 2^2 \alpha^4 [\mathcal{Q}(p_1 - p'_1, p_2 - p'_2, p_3 - p'_3) + 35 \text{ terms}]
\end{aligned} \tag{2.33}$$

from which, after imposing that the result be finite, we get

$$\begin{aligned}
C &= \lambda^2 \frac{7}{48\pi^2} \frac{1}{\epsilon} - \lambda \alpha^2 \left[\frac{33}{16\pi^2} \right] \frac{1}{\epsilon} + \alpha^4 \frac{72}{2\pi^2} \frac{1}{\epsilon} \\
& - \lambda^3 \left[\frac{582 + 57\pi^2 - 1092\gamma}{2^{14}\pi^4} \right] \frac{1}{\epsilon} + \lambda^3 \left[\frac{49}{2^8 3^2 \pi^4} \right] \frac{1}{\epsilon^2},
\end{aligned} \tag{2.34}$$

The term proportional to α^4 in the above expression shows that the renormalization of λ is not multiplicative, a fact that will lead to an interesting effect in the renormalization group equations. In the next section, results (2.22), (2.34) and $B = 0$ will be used to determine the renormalization group parameters.

III. RENORMALIZATION GROUP ANALYSES

Let us start by verifying the that the CS coupling does not run. Equation (2.5) is

$$\alpha_0 = \alpha \mu^\epsilon \frac{(1 + E)}{(1 + A)(1 + B)}. \tag{3.1}$$

As we have seen in the last section, $B = 0$ and, as consequence of the Ward Identities, we also have $E = A$. Thus (3.1) reduces to

$$\alpha_0 = \alpha \mu^\epsilon, \tag{3.2}$$

from which, in the way of [8] we get

$$0 \equiv \mu^{1-\epsilon} \frac{d\alpha_0}{d\mu} = \epsilon \alpha + \mu \frac{d\alpha}{d\mu}, \tag{3.3}$$

and therefore

$$\beta_\alpha = \mu \left. \frac{d\alpha}{d\mu} \right|_{\epsilon \rightarrow 0} \rightarrow 0, \tag{3.4}$$

showing that α does not run under a rescaling of μ or the momenta of the Green function. A similar result was get in [5] for a model of a scalar field interacting with a non Abelian CS field. These results extend to massless matter, the result of the theorem of Coleman-Hill [9].

For calculating β_λ we start with equation (2.6):

$$\lambda_0 = \mu^{2\epsilon} \frac{\lambda + C}{(1 + A)^3} = \mu^{2\epsilon} (\lambda + C - 3A + \dots) \quad (3.5)$$

By substituting (2.22) and (2.34) in this equation we get

$$\lambda_0 = \mu^{2\epsilon} \left(\lambda + \frac{\lambda_1(\alpha, \lambda)}{\epsilon} + \dots \right), \quad (3.6)$$

where

$$\lambda_1(\alpha, \lambda) = a(\lambda^2 - c\alpha^2\lambda + d\alpha^4 - b\lambda^3) \quad (3.7)$$

with

$$a = \frac{7}{48\pi^2} = 0.01478, \quad (3.8)$$

$$b = \frac{1}{2^{10}7\pi^2}(1744 + 171\pi^2 - 3276\gamma) = 0.0218, \quad (3.9)$$

$$c = \frac{120}{7} = 17.1429, \quad (3.10)$$

and

$$d = \frac{1728}{7} = 246.86. \quad (3.11)$$

From (3.6) we have

$$\begin{aligned} 0 &= \mu^{1-2\epsilon} \frac{d\lambda_0}{d\mu} \\ &= 2\epsilon \left(\lambda + \frac{\lambda_1}{\epsilon} + \dots \right) + \left(\mu \frac{\partial \lambda}{\partial \mu} + \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial \lambda_1}{\partial \lambda} \frac{1}{\epsilon} + \mu \frac{\partial \alpha}{\partial \mu} \frac{\partial \lambda_1}{\partial \alpha} \frac{1}{\epsilon} + \dots \right), \end{aligned} \quad (3.12)$$

and using (3.3) we get

$$\begin{aligned} \beta_\lambda &= \mu \frac{\partial \lambda}{\partial \mu} \\ &= \left(\alpha \frac{\partial}{\partial \alpha} + 2\lambda \frac{\partial}{\partial \lambda} - 2 \right) \lambda_1(\alpha, \lambda) - 2\lambda\epsilon \\ &= 2a(\lambda^2 - c\lambda\alpha^2 + d\alpha^4 - 2b\lambda^3) \quad (\text{for } \epsilon \rightarrow 0) \end{aligned} \quad (3.13)$$

Up to 2 loops (terms in λ^2 , $\lambda\alpha^2$ and α^4) this result qualitatively coincides with that of [4] for this same model. It does not, however, coincide with the result of [14] (we will discuss this fact in the conclusions). As can be seen from (3.10), the contribution of the 4 loops graphs (term in λ^3) is small and will not qualitatively change the results for β_λ .

Making $\alpha = 0$ we go to the pure $(\phi^\dagger\phi)^3$ model. In this case β starts at zero for $\lambda = 0$ and increases monotonically with λ [1]. The model presents an infrared (IR) fix point at $\lambda = 0$.

For $\alpha \neq 0$ a drastic change occurs. In this case β starts at $(4ad\alpha^4) > 0$ for $\lambda = 0$ and never vanishes in the perturbative range of the two coupling constants. A similar behavior of the β function, already in one loop order, is shown in the Coleman-Weinberg model (CW) [15] in (3+1) dimensions. There, a dynamical symmetry breakdown occurs and masses are generated for the two fields. In [14] the effective potential was calculated in two loops and a breakdown of symmetry was also shown to appear. We would like to stress that our results for Γ^2 and Γ^6 are compatibles with that conclusion. The $\Gamma^2(v)$ for the displaced field $\psi = \phi - v$, where v is a constant with dimension $(mass)^{1/2}$, would be written, in terms of the functions that we calculated for ϕ , as a series of the form $\Gamma^2(v) = \Gamma^2 + (v^2/2)\Gamma^4 + (v^4/4!)\Gamma^6 + \dots$. As can be seen from the graphs proportional to α^4 in figure 5, Γ^6 receives a constant (independent of p) finite contribution. As consequence, $\Gamma^2(v)$ will have a singularity displaced to some non null value of p^2 , compatible with a non null dynamically generated mass for ϕ .

The anomalous dimensions of the fields A_μ and ϕ are given by

$$\gamma_A = \frac{1}{2} \frac{\mu}{Z_A} \frac{dZ_A}{d\mu} \quad (3.14)$$

$$\gamma_\phi = \frac{1}{2} \frac{\mu}{Z_\phi} \frac{dZ_\phi}{d\mu} . \quad (3.15)$$

As shown in section II, $Z_A = 1 + B = 1$ and so $\gamma_A = 0$. By writing

$$Z_\phi = 1 + A = 1 + \frac{a_1(\alpha, \lambda)}{\epsilon} + \dots \quad (3.16)$$

where a_1 is given in (2.22) we can write (3.15) in the form

$$2 \left(1 + \frac{a_1}{\epsilon} + \dots \right) \gamma_\phi = \mu \frac{\partial \lambda}{\partial \mu} \frac{\partial a_1}{\partial \lambda} \frac{1}{\epsilon} + \mu \frac{\partial \alpha}{\partial \mu} \frac{\partial a_1}{\partial \alpha} \frac{1}{\epsilon} + \dots , \quad (3.17)$$

and using (3.3) and (3.13) we get

$$\gamma_\phi = -\lambda \frac{\partial a_1}{\partial \lambda} - \frac{\alpha}{2} \frac{\partial a_1}{\partial \alpha} . \quad (3.18)$$

By substituting a_1 , from (2.22), in (3.18) we have

$$\gamma_\phi = -\frac{7}{48\pi^2} \alpha^2 + \frac{1}{3^2 \cdot 2^{12} \pi^4} \lambda^2 . \quad (3.19)$$

The contribution in α^2 qualitatively agrees with the result of [4]. The term in λ^2 comes from 4 loops graphs (not calculated in [4]) and is very small compared to the term in α^2 . It can be seen from (3.18), that the scalar field dimension, $D_\phi = \frac{1}{2} + \gamma_\phi$, decreases with the coupling to the CS field. As it is well known, in non perturbative approach in quantum mechanics, the coupling of matter fields to a CS field, changes the spin and statistics of the matter fields, driving bosons into anyons and also, for strong enough coupling, into fermions. Based on these results, there is a conjecture in the literature [10] that, even in perturbative quantum field approach (in which the strength $\alpha \ll 1$) the dimension of a boson coupled to a CS should receive an increase in the direction of the fermion dimension $d_\psi = 1$ (for

the corresponding problem of fermions a decrease in the direction of the boson dimension should be expected). As shown in (3.19) this conjecture is not realized: the coupling to the CS field works in the direction of decreasing the dimension of ϕ .

To get a bit farther in testing this conjecture, we have also looked at the anomalous dimensions of the composite operators $[(\phi^\dagger\phi)^n]$, where n is an integer number. As we are mainly interested in the effect of the coupling of the boson to the CS field, to simplify the calculations, we will restrict the analyze to $\lambda = 0$. In terms of monomials of ϕ this composite operator can be written [12]

$$[(\phi^\dagger\phi)^n] = Z_n(\phi^\dagger\phi)^n + Z_{n-1}^0(\phi^\dagger\phi)^{n-1} + Z_{n-2}^2(\phi^\dagger\phi)^{n-2}(\phi^\dagger\partial^2\phi) + \dots \quad (3.20)$$

Determination of the Z_m^i ($m \leq n$) require the calculation of the divergent parts of the 2m scalar field 1PI vertex functions with the insertion of one integrated composite operator

$$\Gamma_{[(\phi^*\phi)^n]}(x_1, \dots, y_m) = \int d^3z < T[(\phi^\dagger\phi)^n](z)\phi(x_1)\dots\phi(x_m)\phi^\dagger(y_1)\dots\phi^\dagger(y_m) > , \quad (3.21)$$

or, in momentum space, the $\Gamma_{[(\phi^*\phi)^n]}(p_1, \dots, p_{2m})$ function with zero momentum q entering through the special vertex $[(\phi^\dagger\phi)^n]$. Up to order α^2 , the divergent graphs contributing to $\Gamma_{[(\phi^*\phi)^n]}(p_1, \dots, p_{2n})$ are shown in figure 6. In figure 7 we draw some of the graphs that could contribute to $\Gamma_{[(\phi^*\phi)^n]}(p_1, \dots, p_{2(n-1)})$. Diagrams in figure 7, are in fact all nulls, what imply that the renormalization parameters Z_{n-1}^i also vanish. The same can be shown to be true for all Z_m^i which any $m < n$. So, the right side of (3.21) reduces to only the first monomial and $[(\phi^\dagger\phi)^n]$ does not mix with other operators (mixing will however appear if we consider $\lambda \neq 0$). Its renormalization only requires the calculation of Z_n , what means to calculate the divergent parts of the graphs in figure 6. The involved Feynman integrals are the $\mathcal{G}(p, q)$ and $\mathcal{H}(p)$ from figure 5. By writing $Z_n = 1 + A_n$ we have

$$\Gamma_{[(\phi^*\phi)^n]}(p_1, \dots, p_{2n}) = (n!)^2[A_n - (4n^2 - 2n)\alpha^2\mathcal{G} - 2in^2\alpha^2\mathcal{H}] + \text{finite graphs} , \quad (3.22)$$

and we have for A_n

$$\begin{aligned} A_n &= \text{DivPart} \{ (4n^2 - 2n)\alpha^2\mathcal{G} + 2in^2\alpha^2\mathcal{H} \} \\ &= -\frac{4n^2 - n\alpha^2}{16\pi^2} \frac{1}{\epsilon} , \end{aligned} \quad (3.23)$$

where "DivPart" stands for keeping only the divergent part of the following expression.

With these results for Z_m and (2.22) for Z_ϕ , the equation (3.20) rewritten in terms of the unrenormalized (see also (2.2)) field ϕ_0 , becomes

$$[(\phi^\dagger\phi)^n] = Z_{cn}^{-1}(\phi_0^\dagger\phi_0)^n , \quad (3.24)$$

where

$$Z_{cn} = (Z_n)^{-1}(Z_\phi)^n = 1 + \frac{a_{cn}(\alpha)}{\epsilon} + \dots , \quad (3.25)$$

and

$$a_{cn}(\alpha) = \frac{\alpha^2}{4\pi^2} \left(n^2 + \frac{n}{3} \right) . \quad (3.26)$$

By deriving the two sides of (3.24) with respect to μ and remembering that ϕ_0 is independent of μ we have

$$\mu \frac{d}{d\mu} [(\phi^\dagger \phi)^n] = -\gamma_{cn} [(\phi^\dagger \phi)^n], \quad (3.27)$$

where

$$\gamma_{cn} = \frac{\mu}{Z_{cn}} \frac{dZ_{cn}}{d\mu}, \quad (3.28)$$

is the anomalous dimension of the composite operator. Going through the same steps that leads (3.16) to (3.18) we get

$$\gamma_{cn} = -\alpha \frac{da_{cn}}{d\alpha} = -\frac{\alpha^2}{2\pi^2} \left(n^2 + \frac{n}{3} \right). \quad (3.29)$$

The dimension of the composite operator $[(\phi^\dagger \phi)^n]$ becomes

$$D_{[(\phi^\dagger \phi)^n]} = n - \frac{\alpha^2}{2\pi^2} \left(n^2 + \frac{n}{3} \right). \quad (3.30)$$

This result is in disagreement with [6]. Their calculation seems to miss the contribution of the second graph in our figure 6. But it is not this fact what makes the major difference. Our counting of the combinatorial factors of the graphs in figure 6, gives a term proportional to n^2 (besides the term in n), different from theirs which is only proportional to n .

No matter if the composite operator is super-renormalizable ($n < 3$), renormalizable ($n = 3$) or non-renormalizable ($n > 3$), the effect of the coupling to the CS field is to lower its dimension. Nevertheless, the lowest non-renormalizable operator, $(\phi^\dagger \phi)^4$, with effective dimension: $D_4 = 4 - \frac{52}{6\pi^2} \alpha^2$ will never, in the perturbative regime, be driven to be renormalizable. Yet, due to the quadratic dependence of the anomalous dimension on n , given any $\alpha \ll 1$, the operators $[(\phi^\dagger \phi)^n]$ with n bigger than $n_c \simeq \frac{2\pi^2}{\alpha^2} - \frac{10}{3} \gg 1$ have their operator dimensions driven to values smaller than 3.

To finish this section, let us look at the renormalization group equations for the $\Gamma_{(2n)}(p, \lambda, \alpha, \mu)$ functions (p is a short for the $2n$ external momenta). As the 4 loops contributions are very small we will restrict the analyses to 2 loops. As β_α and γ_A are null we have the renormalization group equation

$$\left(\mu \frac{\partial}{\partial \mu} + \beta_\lambda \frac{\partial}{\partial \lambda} - 2n\gamma_\phi \right) \Gamma_{(2n)}(p, \lambda, \alpha, \mu) = 0. \quad (3.31)$$

The solution of this equation can be written as

$$\Gamma_{(2n)}(p, \lambda, \alpha, \mu) = \Gamma_{(2n)}(p, \bar{\lambda}, \alpha, \mu s_{\bar{\lambda}, \lambda}) s_{\bar{\lambda}, \lambda}^{2n\gamma_\phi} \quad (3.32)$$

where we used the fact that up to two loops, $\gamma_\phi = -\frac{7}{48\pi^2} \alpha^2$ does not change with s . In the above equation, $s_{\bar{\lambda}, \lambda}$ stands for the solution of

$$\begin{aligned} s \frac{d}{ds} \bar{\lambda} &= \beta_\lambda(\bar{\lambda}) \\ &= 2a(\bar{\lambda}^2 - c\alpha^2 \bar{\lambda} + d\alpha^4), \end{aligned} \quad (3.33)$$

with the condition $\bar{\lambda}(s=1) = \lambda$, that is :

$$\begin{aligned} s_{\bar{\lambda}\lambda} &= \exp \left(\frac{1}{af\alpha^2} \left[\tan^{-1} \left(\frac{2\bar{\lambda}}{f\alpha^2} - \frac{c}{f} \right) - \tan^{-1} \left(\frac{2\lambda}{f\alpha^2} - \frac{c}{f} \right) \right] \right) \\ &\cong \exp \left(\frac{2.86}{\alpha^2} \left[\tan^{-1} \left(\frac{\bar{\lambda}}{12\alpha^2} - 0.71 \right) - \tan^{-1} \left(\frac{\lambda}{12\alpha^2} - 0.71 \right) \right] \right), \end{aligned} \quad (3.34)$$

where $f = (4d - c^2)^{1/2}$. As β_λ is non null for $\lambda = 0$ (for $\alpha \neq 0$) this equation is well defined if we choose $\lambda = 0$. With this choice in (3.34) we can write

$$\Gamma_{(2n)}(p, \bar{\lambda}, \alpha, \mu) = \Gamma_{(2n)}(p, 0, \alpha, \mu s_{\bar{\lambda}0}^{-1}) s_{\bar{\lambda}0}^{-2n\gamma_\phi}. \quad (3.35)$$

This equation shows that, up to two loops, the $\Gamma_{(2n)}$ functions of the model defined by Lagrangian (2.7), can be get from the corresponding $\Gamma_{(2n)}$ for the model where only the interaction term with the A_μ field is present, or what is equivalent, from the calculation of the sub set of diagrams contributing to $\Gamma_{(2n)}$, which only involves the interaction vertex with the A_μ field. A short inspection of the CW [15] results, shows that a similar fact is also true for that model (at least in one loop).

IV. CONCLUSIONS

The coupling to the CS field lowers the dimension of ϕ and of $(\phi^\dagger\phi)^n$. This goes in the opposite direction of the conjecture that the transmutation of the boson into anyon (due to the coupling to the CS field) should be signaled by the dimension of these operators to increase in the direction of the canonical dimension of a fermion field ψ and their composite operators $(\psi^\dagger\psi)^n$, respectively.

In the present paper, as in previous calculations in the literature, the function β_α and the anomalous dimension of the CS field are shown to vanish; the CS coupling constant α does not run with the change of the energy scale. The function β_λ instead, shows a drastic change in the presence of the CS field. From an IR trivial fix point for the pure $\lambda(\phi^*\phi)^3$ interaction, the model is driven, to a phase in which no fix point appears for β_λ , in a behavior similar to that of β_λ for the model of Coleman-Weinberg [15].

In [14], the renormalization group functions were calculated up to 2 loops, although their main aim was to study the effective potential and dynamical symmetry breakdown. The model of [14], defined by their Lagrangian (2.1) can be made to coincide with ours by deleting their $\lambda(\phi^*\phi)^2$ interaction and the $m^2(\phi^*\phi)$ mass term, that is, by making their λ and m zero. Considering also, that their coefficient, ν , of the $(\phi^*\phi)^3$ interaction, differs from our λ by a factor of $2/5$, what also implies in a $2/5$ factor of difference in the corresponding β functions, their results (equations (10.7-9) and (11.8)), after translated to our notation, can be summarized as: **1.** $\beta_\alpha = 0$ and $\gamma_A = 0$. These results are in agreement with our equation (3.4) and the observation below equation (3.15). **2.** $\gamma_\phi = \mathcal{O}(\lambda^2)$, and $\beta_\lambda = 2a\lambda^2 + \mathcal{O}(\lambda^3)$, both independent of α . Our results (3.13) and (3.19) differ from these last ones by terms dependents on the CS coupling α . Their conclusion is that the model has an IR trivial fix point in λ . Ours instead, is that β_λ never vanishes, a result similar to that of CW in a model in which a dynamical symmetry breakdown occurs. A dynamical symmetry breakdown was

also seen in [14] for the present model. Our result for β looks so, in accordance with their result on symmetry breakdown.

The discrepancies between ours and the β function of [14] can be attributed to the different regularization schemes we are using. In [14], the model is regularized through a full dimensional regularization, by extending out of 3D, all the tensor structures (including the definition of the $\epsilon_{\mu\nu\rho}$) that appear in the Feynman graphs. As they conclude, in that method, the renormalizability of the model is only achieved, if an extra regularization, represented by a Maxwell term for the A^μ field (besides the CS one), is introduced. Their method requires, that this extra regularization be dismissed (their parameter “a” taken to zero), only after the continuation back to 3D is made. As can be seen from their results (11.8), some of their β functions become singular, when $a \rightarrow 0$, showing that a better understanding of the structure of the renormalization group equation is still lacking in that method. Also, as discussed in their Section 10, if a regularization directly in 3D (exists and) were used, γ_ϕ and β_λ would be expected to depend also on α .

In this paper we used the Dimensional Reduction regularization scheme, in which all the tensor contractions are first made in 3D and only the remaining scalar Feynman integrals are extended out of 3D. We explicitly verified that this method controls all the UV infinities and preserves the Ward identities (and so, the gauge covariance) up to the order of approximation in which we are working (2 loops in graphs involving the CS propagator and 4 loops in graphs only involving the scalar propagator). Although we can not say that it is a regularization directly in 3D, our results is consistent with the above mentioned discussion in [14].

As a definitive answer to this problem is desirable, we are presently working in a related model, using a direct in 3D version of the BPHZ renormalization method. The preliminary results confirm those of the present paper for the renormalization group functions, together with the dynamical symmetry breakdown got in [14].

To finalize we would like to summarize the results of two previous papers [11], in which we studied the scale behavior of fermions interacting with a CS field. In the first one, a single fermion with its most general 4-fermion (non renormalizable) self interaction $g(\bar{\psi}\psi)^2$ was considered. We saw that, although ψ gets a negative anomalous dimension, making its operator dimension to approach that of a boson, no definite pattern of approach to a bosonic scale behavior due to the interaction with the CS field is seen for composite operators: the super-renormalizable operator $\bar{\psi}\psi$ gets a negative anomalous dimension, but the non-renormalizable operator $(\bar{\psi}\psi)^2$ gets a positive one. In the second paper an extended version of this model with N (small) fermion fields, with their most general 4-fermion interaction: $g(\bar{\psi}\psi)^2 + h(\bar{\psi}\gamma^\mu\psi)^2$ was considered. We studied operators of canonical dimension four. We showed that one of them has positive anomalous dimension, other has a very small negative anomalous dimension and the third one, more interesting from the renormalization view point, has a negative anomalous dimension, making, through a fine tuning of the coupling constants, its operator dimension as close to 3 as wanted . Nevertheless, no general pattern of approach to a bosonic like behavior (negative anomalous dimension), as advanced by the conjecture in the literature, was seen.

APPENDIX A: THE WARD IDENTITIES

The two relations among the counterterms: A to E can be get from the WI among the 1PI 4-linear photon-scalar vertex, $\Gamma_{\mu\nu}$, the trilinear photon-scalar vertex, Γ_μ , and the scalar self energy Γ_2 . In tree approximation they are given by (see figure 1): $\Gamma_{\mu\nu} = 2ie^2\mu^\epsilon g_{\mu\nu}$, $\Gamma_\mu = -ie\mu^{\epsilon/2}(p' + p)_\mu$ and $\Gamma_2 = iAp^2$. It is ease to use that they satisfy the relations

$$q^\mu \Gamma_\mu(q; p, p') = -e\mu^{\frac{\epsilon}{2}} \{ \Gamma_2(p') - \Gamma_2(p) \} , \quad (\text{A1})$$

$$q^\mu \Gamma_{\mu\nu}(q, q'; p, p') = -e\mu^{\frac{\epsilon}{2}} \{ \Gamma_\nu(q'; p' - q', p') - \Gamma_\nu(q'; p, p + q') \} . \quad (\text{A2})$$

As we explicitly verified these relations are, in fact, valid up to 2-loop order. Instead of considering the WI among the sum of all graphs up to 2-loops contributing to each of the 3 vertex functions above, we can take advantage of the fact that they can be separated in sub classes to be seen to be separately related through the the WI (A1) and (A2). As an example consider the graphs (8-a) to (8-h) contributing to Γ_μ and (9-a) and (9-b) contributing to Γ_2 . Let us call $\tilde{\Gamma}_\mu$ the sum of contributions of diagrams (8-a) to (8-q) and $\tilde{\Gamma}_2$ the graph (9-a). Let also \tilde{D} and \tilde{A} be the possible divergent contributions to the counterterms D and A , chosen so as to make finite the sums of graphs in figure 8 and 9, respectively. By using dimensional reduction regularization, and explicitly writing all the Feynman integrands involved, we can verify that

$$q^\mu \{ \tilde{\Gamma}_\mu(q; p, p') - ie\mu^{\frac{\epsilon}{2}}(p' + p)_\mu \tilde{D} \} = -e\mu^{\frac{\epsilon}{2}} \{ (\tilde{\Gamma}_2(p') + ip'^2 \tilde{D}) - (\tilde{\Gamma}_2(p) + ip^2 \tilde{D}) \} . \quad (\text{A3})$$

As \tilde{D} is chosen so as to make the bracket in the left side of these equations finite, the right side is also finite, what implies that: $ip^2 \tilde{D} = -\text{DivPart}\{\tilde{\Gamma}_2(p)\} \equiv ip^2 \tilde{A}$, that is $\tilde{D} = \tilde{A}$. A more direct verification is obtained by explicitly calculating:

$$ie\mu^{\frac{\epsilon}{2}}(p' + p)_\mu \tilde{D} = \text{DivPar}\{\tilde{\Gamma}_\mu(q; q, p')\} \quad (\text{A4})$$

and

$$ip^2 \tilde{A} = -\text{DivPart}\{\tilde{\Gamma}_2(p)\} . \quad (\text{A5})$$

The only really divergent graphs contributing to $\tilde{\Gamma}_\mu$ are (8-a) and (8-g). By going through the calculation of the divergent parts of (8-a) plus (8-g) as exemplified in Appendix B we get

$$\begin{aligned} ie(p' + p)_\mu \tilde{D} &= \text{DivPart} \left\{ (-ie)^3 (ie^2) (i)^3 \frac{12}{3!} \int \mathcal{D}k \int \mathcal{D}k' \varepsilon_{\beta\nu\rho} \frac{k^\rho}{k^2} \varepsilon_{\alpha\mu\gamma} \frac{k'^\gamma}{k'^2} \right. \\ &\quad \times \frac{(2p + k)_\beta (2p + 2k + k')_\alpha (2p + 2k' + k)_\nu}{(p + k)^2 (p + k')^2 (p + k + k')^2} + p \leftrightarrow p' \left. \right\} \\ &= i \frac{e^5}{12\pi^2} \frac{1}{\epsilon} p_\mu + i \frac{e^5}{12\pi^2} \frac{1}{\epsilon} p'_\mu \end{aligned} \quad (\text{A6})$$

that is: $\tilde{D} = \frac{\alpha^2}{12\pi^2} \frac{1}{\epsilon}$.

For \tilde{A} we have

$$\begin{aligned}
ip^2 \tilde{A} &= -\text{DivPart}\{\tilde{\Gamma}(p)\} \\
&= -\text{DivPart}\{\text{Graph (3-b)}\} \\
&= i \frac{e^4}{12\pi^2} \frac{1}{\epsilon} p^2
\end{aligned} \tag{A7}$$

as given by (2.8). So, we have $\tilde{D} = \tilde{A} = \frac{\alpha^2}{12\pi^2} \frac{1}{\epsilon}$.

An example of subset of graphs that match through the 2^d WI are depicted in Fig. 10 and Fig. 11. The identification of $\tilde{E}' = \tilde{D}'$ follows through the same steps as in the example above.

APPENDIX B: FEYNMAN INTEGRALS

To illustrate the method adopted to get the divergent parts of the Feynman integrals that appear in the paper, we will explicitly show as examples the calculation of the diagrams 5.a, 5.b, 5.d and 5.f. Let us start with (5.a). In the figure, $\Delta(k) = i/(k^2 + i\eta)$ as usual, and $\Delta_2(k)$ and $\Delta_3(k)$ stand for the subgraphs formed respectively by 2 and 3 scalar propagators connecting 2 vertices, with total momentum k passing through. Its integration can be done successively one loop at time, first getting Δ_2 and then Δ_3 . $\Delta_2(p)$ is given by

$$\Delta_2(p) = \int \mathcal{D}k \frac{i}{k^2 + i\eta} \frac{i}{(k+p)^2 + i\eta}, \tag{B1}$$

where $\mathcal{D}k = \mu^\epsilon d^{3-\epsilon}k/(2\pi)^{3-\epsilon}$. By introducing a Feynman parameter through the use of the identity

$$\frac{1}{A^\alpha B^\beta} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \frac{x^{\alpha-1}(1-x)^{\beta-1}}{[Ax + B(1-x)]^{\alpha+\beta}}, \tag{B2}$$

the k integration can be done [12] and then the parametric integration [13] to give

$$\Delta_2(p) = -i \frac{(4\pi\mu^2)^{\frac{\epsilon}{2}}}{(4\pi)^{\frac{3}{2}}} \frac{\Gamma^2\left(\frac{1}{2} - \frac{\epsilon}{2}\right) \Gamma\left(\frac{1}{2} + \frac{\epsilon}{2}\right)}{\Gamma(1 - \epsilon)} (-p^2 - i\eta)^{-(\frac{1}{2} - \frac{\epsilon}{2})}. \tag{B3}$$

$\Delta_3(q)$ can be written as

$$\Delta_3(q) = \int \mathcal{D}p \frac{i}{(p+q)^2 + i\eta} \Delta_2(p). \tag{B4}$$

This integration can also be done following the same steps as for $\Delta_2(p)$, after explicitly substituting in this last equation, the expression (B3) for $\Delta_2(p)$. The result is

$$\Delta_3(q) = -\frac{i}{(4\pi)^3} \frac{\Gamma^3\left(\frac{1}{2} - \frac{\epsilon}{2}\right) \Gamma(\epsilon)}{\Gamma\left(\frac{3}{2} - \frac{3\epsilon}{2}\right)} \left(-\frac{4\pi\mu^2}{q^2 + i\eta}\right)^\epsilon. \tag{B5}$$

For 5.d we have

$$\mathcal{W}(q, p) = \int \mathcal{D}k \frac{i}{(k+q)^2 + i\eta} \Delta_2(k+p) \Delta_3(k) , \quad (\text{B6})$$

This integral can be done by first reducing the 3 denominators to a single one, by twice using (B2) to get a single denominator and then doing the k integration [12]. In terms of the 2 remaining Feynman parameters it has the form

$$\mathcal{W}(q, p) = -\frac{1}{(4\pi)^6} \frac{\Gamma(2\epsilon) \Gamma^5\left(\frac{1}{2} - \frac{\epsilon}{2}\right)}{\Gamma(1-\epsilon) \Gamma\left(\frac{3}{2} - \frac{3\epsilon}{2}\right)} \left(-\frac{4\pi\mu^2}{p^2 + i\eta}\right)^{2\epsilon} I_\epsilon(q, p) , \quad (\text{B7})$$

where $I_\epsilon(q, p)$ is given by

$$I_\epsilon(q, p) = \int_0^1 dy (1-y)^{\epsilon-1} f_\epsilon(y) , \quad (\text{B8})$$

and

$$f_\epsilon(y) = \int_0^1 dx x^{-\frac{1}{2} + \frac{\epsilon}{2}} y^{\frac{1}{2} + \frac{\epsilon}{2}} \left\{ \frac{q^2}{p^2} [y^2(1-x)^2 - y(1-x)] + 2\frac{p \cdot q}{p^2} y^2 x(1-x) + yx(yx-1) \right\}^{-2\epsilon} . \quad (\text{B9})$$

$I_\epsilon(q, p)$ has a single pole in ϵ coming from the integration region in the vicinity of $y = 1$. As (B7) already has a factor $\Gamma(2\epsilon)$ the integral (B6) will present both a single and a double pole in ϵ . To separate their contributions we must calculate the first 2 terms (single pole and the ϵ independent term) of the Laurent expansion of $I_\epsilon(q, p)$. We have

$$\begin{aligned} I_\epsilon(q, p) &= I_{1\epsilon}(q, p) + I_{2\epsilon}(q, p) \\ &= \frac{A_1}{\epsilon} + (B_1 + B_2) + (C_1 + C_2)\epsilon + \dots , \end{aligned} \quad (\text{B10})$$

where

$$\begin{aligned} I_{1\epsilon}(q, p) &= \int_0^1 dy (1-y)^{\epsilon-1} f_\epsilon(1) \\ &= \frac{A_1}{\epsilon} + B_1 + C_1\epsilon + \dots , \end{aligned} \quad (\text{B11})$$

$$\begin{aligned} I_{2\epsilon}(q, p) &= \int_0^1 dy (1-y)^{\epsilon-1} (f_\epsilon(y) - f_\epsilon(1)) \\ &= B_2 + C_2\epsilon + \dots , \end{aligned} \quad (\text{B12})$$

where A_1 , B_1 and B_2 are still to be determined. B_2 is given by

$$B_2 = I_{20}(q, p) = \int_0^1 dy (1-y)^{-1} \int_0^1 dx (y^{\frac{1}{2}} - 1) x^{-\frac{1}{2}} = 4(-1 + \log 2) . \quad (\text{B13})$$

A_1 and B_1 come from

$$\begin{aligned}
I_{1\epsilon}(p, q) &= \int_0^1 dy (1-y)^{\epsilon-1} \int_0^1 dx x^{\frac{\epsilon}{2}-\frac{1}{2}} (x^2-x)^{-2\epsilon} \left\{ \frac{(q-p)^2}{(p^2)} \right\}^{-2\epsilon} \\
&= (-1)^{-2\epsilon} \left(\frac{p^2}{(q-p)^2} \right)^{2\epsilon} \frac{\Gamma\left(\frac{1}{2}-\frac{3\epsilon}{2}\right) \Gamma(1-2\epsilon) \Gamma(\epsilon)}{\Gamma\left(\frac{3}{2}-\frac{7\epsilon}{2}\right) \Gamma(1+\epsilon)}.
\end{aligned} \tag{B14}$$

The results are $A_1 = 2$ and $B_1 = 2 \left\{ 5 - 2 \log 2 - \frac{7\gamma}{2} + 2 \log \left(-\frac{p^2}{(p-q)^2} \right) \right\}$. Multiplying the Laurent expansion of $I_\epsilon(p, q)$ by the Laurent expansion of the multiplying factor in (B7) we get

$$\mathcal{W}(q, p) = -\frac{1}{2^{11}\pi^4} \frac{1}{\epsilon^2} - \frac{1}{2^{10}\pi^4} \left\{ 4 - \frac{11}{4}\gamma + \log \left(-\frac{4\pi\mu^2}{(p-q)^2} \right) \right\} \frac{1}{\epsilon} + \text{finite part}. \tag{B15}$$

Let us go to (5.b). The sub diagram $D_2(k)$ is given by

$$D_2(k) = \int \mathcal{D}q \varepsilon^{\mu\nu\lambda} \frac{q_\lambda}{q^2 + i\eta} \varepsilon_{\nu\mu\rho} \frac{(q+k)^\rho}{(q+k)^2 + i\eta}. \tag{B16}$$

After contracting the tensors in $(2+1)$ dimension we are left with the (finite) integral

$$\begin{aligned}
D_2(k) &= -2 \int \mathcal{D}q \frac{k \cdot q}{[q^2 + i\eta][(q+k)^2 + i\eta]} \\
&= -\frac{i}{8} \frac{(4\pi\mu^2)^{\frac{\epsilon}{2}}}{(-k^2 - i\eta)^{-\frac{1}{2}+\frac{\epsilon}{2}}},
\end{aligned} \tag{B17}$$

where ϵ was made zero whenever possible. Graph (5.b) is given by

$$\mathcal{G}(p_1, p_2) = \int \mathcal{D}k \frac{i}{(p_1+k)^2 + i\eta} \frac{i}{(p_2+k)^2 + i\eta} D_2(k). \tag{B18}$$

This integral is logarithmically divergent and their residue is independent of p_1 and p_2 . To get this residue it is sufficient to calculate it for $p_2 = -p_1$

$$\mathcal{G} \big|_{p_2=-p_1} = \int \mathcal{D}k \frac{i}{[-(p+k)^2 - i\eta]^2} \frac{1}{(-k^2 - i\eta)^{-\frac{1}{2}+\frac{\epsilon}{2}}}, \tag{B19}$$

where whenever possible we have put $\epsilon = 0$. After introducing a Feynman parameter through (B2) and integrating in k we get

$$\mathcal{G} \big|_{p_2=-p_1} = -\frac{1}{2^5\pi^2} \Gamma(\epsilon) (-p_1^2)^{-\epsilon} = -\frac{1}{32\pi^2} \frac{1}{\epsilon} + \dots \tag{B20}$$

Contribution of diagram (5.f) is given by

$$\mathcal{H}(p) = -i \int \mathcal{D}q \mathcal{D}k \varepsilon^{\mu\nu\rho} \frac{(p+q)_\rho}{(p+q)^2} \varepsilon^{\alpha\beta\lambda} \frac{q_\lambda}{q^2} \frac{g_{\nu\alpha}(2k+p-q)_\mu (2k-q)_\beta}{(k+p)^2 (k-q)^2 k^2}. \tag{B21}$$

or

$$\mathcal{H}(p) = -2i\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\frac{\mu^\epsilon}{(2\pi)^d}\int\mathcal{D}q\frac{1}{(p+q)^2q^2}I_{\beta\mu\lambda\rho}(q), \quad (\text{B22})$$

where

$$I_{\beta\mu\lambda\rho}(q) = \int d^dk \frac{2k_\beta k_\mu (q_\nu p_\rho + q_\nu q_\rho) + k_\beta (q_\nu q_\rho p_\mu - q_\nu q_\mu p_\rho)}{(k+p)^2(k-q)^2k^2}. \quad (\text{B23})$$

Using the identity

$$\frac{1}{ABC} = 2 \int_0^1 y dy \int_0^1 dx \frac{1}{[C(1-y) + y(Ax + B(1-x))]^3} \quad (\text{B24})$$

and doing the k integration we get

$$\begin{aligned} \mathcal{H}(p) = & -\frac{2\Gamma\left(2-\frac{d}{2}\right)\mu^\epsilon}{2^d\pi^{\frac{d}{2}}}\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\int_0^1 y dy \int_0^1 dx \frac{1}{[-a']^{3-\frac{d}{2}}}\int\mathcal{D}q\frac{1}{(p+q)^2q^2} \\ & \times \left[\frac{q_\nu q_\mu p_\rho p_\beta xy[(4-d)y(x-1)+1] + q_\nu q_\rho p_\mu p_\beta xy[(4-d)xy-1]}{[-q^2 - 2q\cdot p\frac{b'}{a'} - p^2\frac{c'}{a'}]^{3-\frac{d}{2}}} \right. \\ & \left. + \frac{a'g_{\beta\mu}(q_\nu p_\rho + q_\nu q_\rho)}{[-q^2 - 2q\cdot p\frac{b'}{a'} - p^2\frac{c'}{a'}]^{2-\frac{d}{2}}} \right] \end{aligned} \quad (\text{B25})$$

with $a' = y(x-1)[y(x-1)+1]$, $b' = xy^2(x-1)$ and $c' = xy(xy-1)$. The only divergent term is the last monomial in the square bracket of (B25). We can so, write

$$\mathcal{H}(p) = \frac{\Gamma\left(2-\frac{d}{2}\right)\mu^\epsilon}{2^{d+1}\pi^{\frac{d}{2}}}\varepsilon^{\mu\nu\rho}\varepsilon^{\alpha\beta\lambda}g_{\nu\alpha}\int_0^1 dy y \int_0^1 dx \frac{1}{[-a']^{2-\frac{d}{2}}}\frac{\mu^\epsilon}{(2\pi)^d}I_{DivPar}(p) + \text{fin parts} \quad (\text{B26})$$

where

$$I(p) = \int d^dq \frac{q_\nu q_\rho}{(p+q)^2q^2[-q^2 - 2q\cdot p\frac{b'}{a'} - p^2\frac{c'}{a'}]^{2-\frac{d}{2}}}. \quad (\text{B27})$$

By reducing the denominators through the use of the identity

$$\frac{1}{A^\alpha B^\beta C^\gamma} = \frac{\Gamma(\alpha+\beta+\gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}\int_0^1 dz \int_0^z dt \frac{t^{\gamma-1}(z-t)^{\beta-\alpha}}{[A+(B-A)z+(C-B)t]^{\alpha+\beta+\gamma}}. \quad (\text{B28})$$

and doing the integration in q we get for the divergent part

$$I_{DivPart}(p) = -i\frac{g_{\lambda\rho}}{2^{d+1}\pi^{\frac{d}{2}}[p^2]^\epsilon}\frac{\Gamma(\epsilon)}{\Gamma\left(2-\frac{d}{2}\right)}\int_0^1 dz \int_0^z dt t^{1-\frac{d}{2}}[a''-b'']^{d-3}, \quad (\text{B29})$$

where $a'' = 1-z+\frac{b'}{a'}t$ and $b'' = 1-z+\frac{c'}{a'}t$. By inserting (B29) in (B27), expanding in ϵ and doing the parametric integrations we get

$$\mathcal{H} = \frac{i}{16\pi^2}\frac{1}{\epsilon} + \text{finite parts}. \quad (\text{B30})$$

ACKNOWLEDGEMENTS

This work was partially supported by the Brazilian agencies: Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Coordenadoria de Aperfeiçoamento de Pessoal de Ensino Superior (CAPES) and by Fundação de Amparo a Pesquisa do Estado de São Paulo (FAPESP).

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Figure Captions

- Figure 1. Feynman rules in the Landau gauge.
- Figure 2. Divergent diagrams contributing to the CS 2 point function.
- Figure 3. Divergent diagrams contributing to the scalar field 2 point function.
- Figure 4. Divergent contributions to the scalar 6 point function. Three others, not drawn, diagrams similar to 4.n, 4.o and 4.p, but with the sense of all external lines reversed must also be considered.
- Figure 5. Representation of the divergent integrals that appear in the diagrams of Figure 4.
- Figure 6. Divergent contributions to $\Gamma_{[(\phi^*\phi)^n]}(2n)$, that is, the $2n$ point function with one insertion of the composite operator $[(\phi^*\phi)^n]$.
- Figure 7. Some possible contributions to $\Gamma_{[(\phi^*\phi)^n]}(2(n-1))$, the $2(n-1)$ point function with one insertion of the composite operator $[(\phi^*\phi)^n]$.
- Figure 8. An example of a family of diagrams contributing to $\Gamma_\mu(q; p, p')$.
- Figure 9. The diagrams contributing to $\Gamma(p)$, related by the Ward identity, to the family of diagrams in Figure 8.
- Figure 10. An example of family of diagrams contributing to $\Gamma_{\mu\nu}(q, q'; p, p')$.
- Figure 11. The diagrams contributing to Γ_μ , related by the Ward identity to the family of diagrams in Figure 10.